

The Restoration of the Electroweak Symmetry at High Temperature for Little Higgs

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Abstract

In this letter, we show that the electroweak symmetry is restored at high temperature for Little Higgs (LH), when including dominant higher order thermal corrections, that are consequence of the non-linear nature of the scalar sector. This leads us to suggest that the LH requires a UV completion above the scale $\Lambda \lesssim f$.

A promising approach had been proposed to solve the hierarchy problem, where the Higgs field manifests as a pseudo-Goldstone boson since 3 decades ago [1]. This idea was revived by the development of the Little Higgs (LH) [2]. The quadratic corrections (QCs) to the Higgs mass are canceled by the contributions of new introduced fields as in supersymmetry. In these models bosonic corrections cancel each other; and similarly for fermions. This is imposed by the presence of such a global symmetry that is broken spontaneously by a new scalar vev ($f \sim 1\text{-}10 \text{ TeV}$); in a smart way (collective breaking), where the Higgs boson mass remains protected from one-loop QCs above a cut-off scale ($\Lambda \sim 4\pi f$), that is estimated using naive dimensional analysis (NDA). The scalar fields correspond to the broken generators of the global symmetry; and the scalar potential is just a combination of effective operators whose gauge and Yukawa origins. The electroweak symmetry breaking (EWSB) is triggered by large one-loop Yukawa contributions [2]. The so-called Littlest Higgs [2], which is based on a spontaneously broken global symmetry $SU(5)$ to $SO(5)$, has been shown to be phenomenologically consistent [3].

However the EW symmetry restoration at temperature seems to be problematic as shown for the Littlest Higgs [4]. This can be understood due to the fact that thermal corrections ($\sim T^2/12$); and quadratic corrections ($\sim 3\Lambda^2/16\pi^2$) are generated at one-loop from the same interactions in any gauge theory. This means that the Higgs thermal corrections will cancel each other also; and the electroweak symmetry will not be restored at high temperature. Indeed, it was shown in [4], that above such critical temperature $T_c \sim f$, the thermal corrections become negative and the absolute minimum ($\langle h \rangle \neq 0$), gets deeper at higher temperatures instead of being relaxed to zero.

In this work, taking the Littlest Higgs as an example, we try to understand this unusual behavior of the effective scalar potential. We will check whether this feature is intrinsic for Little Higgs or just a consequence of incomplete computations, and therefore the symmetry can be restored at high temperatures.

First let us briefly review the Littlest Higgs model. It is based on an $SU(5)/SO(5)$ nonlinear sigma model, where the $SU(5)$ symmetry is spontaneously broken down to $SO(5)$ by a vacuum expectation value of a 5×5 symmetric matrix scalar field, Σ_0 . The $SU(5)$ subgroup $(SU(2) \times U(1))_1 \times (SU(2) \times U(1))_2$; is gauged, its diagonal subgroup being the SM electroweak group $SU(2)_L \times U(1)_Y$, and the axial generators correspond to new heavy gauge bosons. The global $SU(5)$ symmetry breaking results 14 Goldstone bosons: 4 are identified to be the Standard Model (SM) Higgs doublet, 6 as complex triplet and 4 as the Goldstone bosons that give masses to the new heavy gauge bosons. These scalar degrees of freedom are represented in the nonlinear representation as

$$\Sigma = e^{2i\pi_k X_k/f} \Sigma_0, \quad (1)$$

where π_k are the 14 Goldstone bosons, X_k are the $SU(5)$ broken generators, and f is the value of the vev that breaks the global symmetry. The quark sector also involves a new heavy singlet quark U , with two Yukawa

couplings $\lambda_{1,2}$, which related to standard Yukawa coupling as $1/\lambda_t^2 = 1/\lambda_1^2 + 1/\lambda_2^2$. Similar relations hold for the gauge couplings $g_{1,2}$ and $g'_{1,2}$: $1/g^2 = 1/g_1^2 + 1/g_2^2$ and $1/g'^2 = 1/g'_1 + 1/g'_2$.

The scalar potential is a summation of effective mass operators due to the gauge and Yukawa interactions, which is given a la Coleman-Weinberg by [5]

$$V_{CW}(\Sigma) = a_V \text{Tr} [M_g^2] + a_F \text{Tr} [M_Y \cdot M_Y^\dagger], \quad (2)$$

where $a_V, a_F \sim \mathcal{O}(1)$ are unknown parameters associated with these effective operators; their values depend on the UV completion of the theory. The theory ground state is stable in both of h - and ϕ -directions if $\Theta = a_V [g_1^2 + g_2^2 + g_1'^2 + g_2'^2]/4 - a_F \lambda_1^2/2 > 0$, and the EW symmetry is not broken, where h and ϕ are the neutral components of the doublet and triplet, respectively. But when including the one-loop corrections (especially the Yukawa's), the EW symmetry gets broken and the SM fields acquire masses [2].

In Ref. [4], it was shown in figures (1) and (2); how the EW symmetry gets broken in a slight way after including the one-corrections. While in figure (3), it was shown how thermal corrections help to relax the minimum to zero especially for low temperature values ($T \leq 0.44f$); but the maximum of the potential (at $h = \pi f/\sqrt{2}$) is getting down when increasing the temperature until becomes degenerate together with the absolute minimum at a critical temperature $T_c \sim 0.96f$. Above this temperature value ($T > f$), this new minimum gets deeper and deeper when increasing temperature. This unusual behavior had lead to the conclusion that symmetry can not be restored at high temperatures in these models.

Before explaining this behavior, let's address the following comment on the EWSB. The realization of a slight EW symmetry (as in figures (1) and (2) of [4]) is practically impossible, unless taking the cut-off value less than the NDA value ($\Lambda < 4\pi f$). This leads to ask questions about the LH breaking scale Λ , and whether the NDA value is the correct one? For the cut-off NDA value, the negative Yukawa one-loop corrections at the absolute minimum, are significantly large with respect to the tree-level potential (or the parameter Θ), which pushes away the Higgs vev from its desired value¹. But when reducing its value until $\Lambda \sim 1.1f$, the Higgs vev is given exactly by the SM value, $v = 246$ GeV, for $f = 1$ TeV²; and the Yukawa contribution is comparable to V_{CW} . This bound is consistent with the suggestion of the cut-off value $\Lambda \leq \Lambda_{NDA}/\sqrt{20} \sim 2.8f$; that is coming from the study of the scalar loops in the Littlest Higgs [6]. Indeed, that is not the only hint to push the cut-off scale below the NDA value, but another bound comes from the unitarity violation where it was suggested that $\Lambda \sim (3-4)f$ [7].

The scalar potential behavior at high temperature that is mentioned in [4] can be understood by estimating the field-dependant masses, especially the Yukawa's, within one period $h \in [0, \sqrt{2}\pi f]$. Naively, the gauge field masses vary below the squared values of the gauge couplings in units of f^2 , and scalar masses vary below a combination of $a_V g_i^2$ and $a_F \lambda_1^2$ in units of f^2 . These masses are significantly small when compared with the two Yukawa eigenmasses, the lightest one lies below λ_t^2 , and the heaviest one lies between λ_2^2 and $\lambda_1^2 + \lambda_2^2$ in units of f^2 . According to these values, the thermal integral [8], that involves (m^2/T^2) as a variable, will not be suppressed for all fields; and the unsuppressed fermionic contributions, that have a large negative multiplicity ($n_t = n_T = -12$), will dominate the thermal scalar potential since they are T^4 -proportional. In other gauge theories, SM as an e.g., most of the fields contributions are suppressed for large values of the scalar field h , but this not the case here because the scalar sector is periodic.

However, one need to comment on the problem of the vacuum misalignment induced by top quark in LH models [9]. In order that the EW vacuum would be the deepest one, some constrains on the gauge couplings are imposed, these constrains become less serious as the cut-off is taken to be less then the NDA value. In the wrong vacuum alignment, that leads to the breaking $(SU(2) \times U(1))^2 \rightarrow (U(1))^2$ instead of $(SU(2) \times U(1))^2 \rightarrow SU(2) \times U(1)$, the Yukawa eigenmasses are of order $\lambda_{1,2}^2$, therefore their thermal contribution to the effective potential will be always less than in the EW vacuum, this means that the EW ground state is always the deepest at finite temperature.³

Then one concludes that this behavior is a consequence of periodicity of the non-linear nature of the scalar representation; in addition to the largeness of negative fermionic contributions to the thermal effective potential, that can not be compensated by other bosonic contributions. But were all the significant contributions taken into account to obtain this behavior?

The nonlinear nature of the scalar sector implies the existence of a new type of interactions like in Fig. 1-a, when expanding the field matrix Σ in powers of $1/f$ in the Lagrangian. These vertices, which are suppressed as $1/f^{n-2}$, could result higher order loop corrections as shown in Fig. 1-b.

These corrections, in Fig. 1-b, are not the only possible contractions of the vertices in Fig. 1-a, but they are the dominant contributions at high temperature, since each scalar loop gives $T^2/12$. Therefore, they lead to a

¹In our work, we have chosen the parameters in a way we can compare our results with those of [4]. One should notice that the authors in [4] have used the $SU(2) \times SU(2) \times U(1)$ model instead the $[SU(2) \times U(1)]^2$ one, but the effect of the missing $U(1)$ is small and then the comparison is still meaningful.

²If we take $f = 2$ TeV ($f = 10$ TeV), the cut-off Λ should be taken $\sim 0.91f$ ($\sim 0.86f$) in order to get $v = 246$ GeV.

³Indeed, a careful study should be performed to check if there are some parameter regions where the wrong vacuum is the deepest one at some temperature values, especially when introducing the corrections (3).

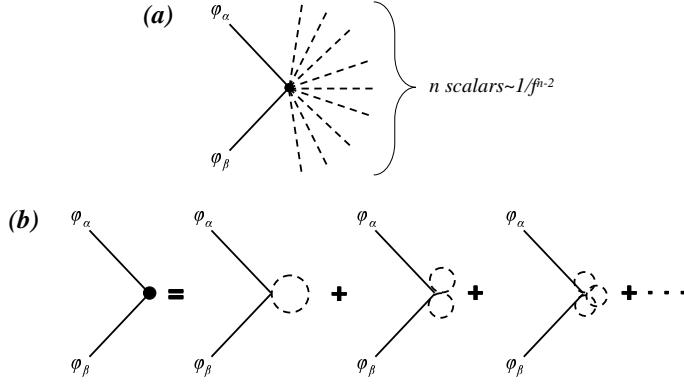


Figure 1: In Fig. a, the fields φ_α and φ_β could be scalars, gauge fields or a fermion-antifermion pair. These novel vertices could result higher order loops corrections to the mass-squared matrix element $M_{\alpha\beta}^2$ as shown in Fig. b.

thermal correction to the field mass-squared matrix element $[\alpha, \beta]$, of the form

$$m^2(T) \sim m^2 + T^2 \sum_n c_n (T^2/f^2)^n, \quad (3)$$

where the zeroth order corresponds to the usual thermal corrections. The computation of the parameters c_n for each mass-squared matrix element tends to determine the vertices of the interactions with scalar degrees of freedom [10], where two temperature regimes could be defined: the first regime where below such temperature $T < f$, only the light degrees of freedom contribute to the c_n parameters, and the second one $T > f$, where all degrees of freedom are in thermal equilibrium. This type of contributions could be very important at temperatures $T \gtrsim f$ if the c_n parameters are not largely suppressed. One can take these novel corrections into account by doing such a resummation with the field-dependant masses in the thermal integral [8], are replaced by thermally corrected masses (3). This is the same way how the daisy contribution [11], was introduced into the effective potential in order to increase the cubic term, and therefore the region of field values where the Higgs and Goldstone bosons self-energies suffer from IR divergences, will be decreased. In the high temperature approximation, this replacement affects mainly the cubic term, and therefore only bosonic fields will be taken into account. Since we have light and heavy particles in our model, and we want to take all of them into account by using the exact formula of the thermal integral [8], we will replace also the fermionic thermally corrected masses to evaluate the effective thermal correction.

Now, the question is how these corrections can help restore the symmetry at high temperatures. As an example, we show in Fig-2 the corrected thermal effective potential at such a temperature at which the unusual behavior described in [4] is supposed to appear, for example $T = 1.7f$, taking into account few order of the masses thermal corrections ($n = 0, 1, 2$) in (3).

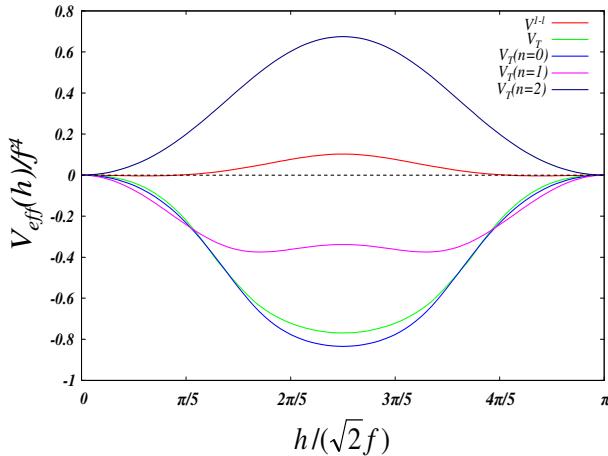


Figure 2: The effective potential at $T=0$, and at $T=1.7f$ computed in the standard way, and using the resummed thermal masses (3) taking into account 1-loop ($n=0$), 2-loop ($n=1$), and 3-loop ($n=2$) corrections.

As it is clear, it is enough to consider only the order ($n = 2$) in (3) to see that the EW symmetry is restored

at this temperature, $T = 1.7f$. Here in this example, we have chosen the cut-off scale to be $\Lambda = 1.3f$. If it is taken to be the NDA value $\Lambda = 4\pi f$, then one needs more corrections ($n > 2$) to show that the symmetry is restored, and for the order $n = 2$, the EW symmetry is restored at $T \simeq 2.854f$. This means that the thermal effective potential behavior shown in [4], is not an intrinsic feature of LH models, but it appears due to the incomplete theory above such a validity scale. This validity scale of the Littlest Higgs model could be estimated from the temperature range, which in below the higher order corrections do not play a significant role in the electroweak dynamics. According to the values of the c_n parameters at the second temperature regime $T > f$, where all particles are in thermal equilibrium, the series (3) could be divergent! [10]. But in the first temperature regime, the c_n parameters values are smaller than the second regime case, and each term in the series (3) is suppressed by an additional T/f power. This naive estimation allows us to suggest that the Little Higgs model is valid up to $\Lambda \lesssim f$, and it is not useful to use the thermal potential at temperatures above this scale. This could open a window to investigate possible realization of a successful baryogenesis at the weak scale within LH models.

In this letter, we have shown that the symmetry non-restoration effect at high temperature for the Littlest Higgs model is due to periodicity of the effective potential with respect to the field value and the largeness of the Yukawa contribution to the thermal effective potential. This behavior disappears when using novel corrections that are coming from the non-linear nature of the scalar representation. We have suggested also a cut-off for the littlest Higgs $\Lambda \lesssim f$, which is consistent with the values suggested from the unitarity study and the scalar loop analysis.

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